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Art. XVI.—*The Psychrometric Formula.*

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(With Plate XLVIII.).

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The formula employed by Regnault for the wet-and-dry bulb hygrometer, or psychrometer, has been generally adopted, viz.:

$$x = f - A\,B\,(t - t'),$$

where x and f are respectively the actual pressure of water-vapour in the air, and the saturation pressure at the temperature t' of the wet-bulb. But Ekholm[1], of Sweden, has suggested modifying this formula by the insertion of a "hygroscopicity factor" η, to allow for a supposed diminution of the maximum pressure at the wet-bulb due to the hygroscopic nature of the material covering it. The formula as modified becomes:—

$$x = \eta f - A\,B\,(t - t'). \quad \eta < 1.$$

Firstly, it may be stated that it seemed to us, on a careful examination of Ekholm's paper, that the grounds for the theory are very slight. Its importance, however, is great enough to justify a careful discussion. It appears to have been suggested by the well-known difficulties attending the use of the psychrometer at temperatures below zero, when the wet-bulb becomes coated with ice. In these circumstances it is necessary to use a value of A different from that used when the water on the bulb is fluid, and observation confirms the

1. Ekholm, "Ueber das Psychrometer," Arkiv. för Mat. Astron. och Fysik, Stockholm, 1908, Bd. 4.

theoretical result that the two values should be in the inverse ratio of the latent heats of ice and water. Regnault noticed that the ice exerts a hygroscopic effect in very humid air, causing condensation on its surface, and other observers since his time have confirmed this. To allow for it, suggestions have been made for a correction of the observed value of either t or t'. Ekholm proceeds a step further by stating that the hygroscopic action of the ice is accompanied by a similar action of the material on which the ice is deposited, this latter action therefore varying with the nature of the material employed. He performed some experiments in which two wet-bulb thermometers coated with different stuffs were enclosed with a dry bulb thermometer in an air-tight vessel; sufficient water was poured in to form a layer on the bottom, and the whole maintained at a temperature below zero. The wet-bulbs, being coated with ice, and in contact with air nearly or completely saturated with water-vapour, received heat by condensation of vapour, and indicated higher temperatures than the dry bulb. This, of course, is due to the fact, unknown to Regnault, that the vapour-pressure of ice is less than that of water at the same temperature. The results of the observations are given as follows:—

Dry Bulb.	Wet Bulb.	
	Cotton.	Wool.
– 4.05	– 3.83	– 3.85
– 4.0	– 3.88	– 3.85
– 4.0	– 3.88	– 3.85
– 4.02	– 3.86	– 3.85
	Cotton.	Linen.
– 3.8	– 3.48	– 3.65
– 3.8	– 3.48	– 3.6
– 3.8	– 3.48	– 3.7
– 3.8	– 3.48	– 3.65
	Cotton.	Silk.
– 3.6	– 3.28	– 3.35
– 3.5	– 3.13	– 3.2
– 3.55	– 3.20	– 3.27

From these the conclusion is drawn that the condensation varies with the material, or that the material exerts a differential hygroscopic effect. Assuming the air contained in the vessel to be saturated, and adopting a value for the constant A given by Regnault for a closed space, the values of η to be used in the formula are calculated—viz.:

Linen - - - -	1.000
Silk - - - -	0.967
Wool - - - -	0.947
Cotton - - - -	0.944

From other considerations the value of η for linen is regarded as more probably 0.9737; in that case the air cannot have been saturated, and the list becomes—

Linen - - - -	0.9737
Silk - - - -	0.940
Wool - - - -	0.921
Cotton - - - -	0.918

Now it is at once evident that these observations are much too few to serve as a basis for any theory. It seems certain that they were not reliable to the hundredth of a degree, and it is only in the case of cotton and linen that any definite difference appears. (Special attention was paid to these two materials, which are the ones most commonly employed in meteorological work.) It is not, however, our intention to criticise these results themselves, but the conclusions which Ekholm, starting from them, reaches for the psychrometer in general. At such low temperatures as this the vapour-pressures are very small, and the instrument consequently insensitive; so we must agree with Ekholm's own remark that systematic observations in cold countries with thermometers graduated in hundredths of a degree are needed before the question of the behaviour of the psychrometer below zero can be satisfactorily discussed and settled.

The modified formula thus obtained is applied by its author to the observations of Regnault[1] and Svensson[2] for tempera-

1. Regnault, "Etudes sur l'hygrométrie," Ann. Chim. Phys., 1853, 3 sér. vol. 37.

2. Svensson, "Untersuchungen des Assmannschen Psychrometers," Meteor. Zeitschr., 1896, p. 201; Bihang till Kongl Svenska Vet.-Akad. Stockholm, 21, 1896.

"Zur Kenntnis des ventilierten Psychrometers," Akad. Abhandlung, Stockholm, 1898. The observations discussed are in the second paper.

tures both above and below zero. Thus Regnault's seventh series was taken with the thermometer covered with cotton muslin and iced. In order to represent his observations with reasonable accuracy, he found it necessary to use two values of A over different ranges—viz., 0.00075 up to 70 per cent., and 0.0013 above 70 per cent. humidity. Ekholm finds that the single formula—

$$x = 0.932 f - 0.000496\ B (t - t')$$

satisfactorily agrees with the whole set of observations, the values of the two constants η and A having been determined by least squares. If, however, Regnault had correctly interpreted his observations by using ice-pressures instead of water-pressures for f, the superiority of the Ekholm formula would not be so striking. The latter employs his own revised tables of vapour-pressures throughout.

Svensson's observations range from about $-3°$ to $+26°$ in dry temperature, and from 5 per cent. to 95.5 per cent. in humidity. For the temperatures above zero, the formula takes the value—

$$x = 0.9737\ f - 0.000596\ B (t - t').$$

Assuming the value of η to be the same whether the material —in this case linen—be covered with water or ice, and reducing the value of A in the theoretical ratio 600 : 680 of the latent heats, the formula appropriate to the lower temperatures becomes

$$x = 0.9737\ f - 0.000526\ B (t - t').$$

This is shown to be in good agreement with the observations, thus apparently confirming the suppositions made.

The same value of η is found to be suitable to Regnault's fourth series of observations, temperatures ranging between 7° and 30°, and a slightly different value, 0.968, to the ninth series, which Regnault regarded as taken in specially unfavourable circumstances.

The argument then rests on the two facts that the modified formula provides a better agreement between the calculated and observed values than the old one; and, secondly, that the value of η which does this is always a proper fraction.

Theoretical Discussion.

The natural answer to these arguments is that they provide no proof whatever of the truth of the new theory, since they are only what we should expect from our knowledge of the conditions and theory of the psychrometer. The formula

$$x = f - A B (t - t')$$

is admittedly only a first approximation, but one which is found in practice to give sufficiently accurate results, provided the value of A is regarded as empirical, i.e., it is to be determined by actual experiment for the special conditions under which the instrument is to be used. It might conceivably be improved in many ways; one that has been tried is the addition of a term $C (t - t')^2$, and this is if anything supported by Svensson's results,[1] since he found the value of A to depend somewhat on that of $t - t'$, being larger for low values of $t - t'$ than for high. But any change in the formula which introduces two arbitrary constants in place of one would, *ceteris paribus*, be likely to increase the accuracy with which it could represent a set of observations. The test of the real value of such an alteration is its correspondence with an actual physical condition, without which we have no certainty that it will apply even approxmately to any other than the particular observations from which it is deduced. In the case in question, we conceive that it has not been proved that the proposed factor η corresponds to an actual definite physical phenomenon, but on the contrary that our observations disprove it.

The second fact, that the value of η as obtained from the observations discussed is consistently less than unity, so far from demonstrating the presence of a hygroscopic action, is also to be expected from theory, as will be seen by consideration of the assumptions underlying the formula. In the usual theory it is supposed that the air is moving past the bulb in such a way that it becomes completely saturated at the temperature t', but does not receive a sensible amount of heat from the surroundings while it is in contact with the bulb. If either of these conditions is not fulfilled, the result will be the same as a diminution of f. And in this connection it is to be

1 See also Rizzo, Nuovo Cimento, Oct. 1897, p. 241.

noticed that Regnault's work was done under conditions which are now recognised to be extremely unfavourable to the right use of the psychrometer. Much of it was carried out in closed spaces where no air-motion was possible, and the effects of radiation from surrounding walls would be a maximum; two of the series discussed by Ekholm, the seventh and the fourth, were taken under these conditions. In other cases the exposure to winds was very different in different directions, and in one case, the ninth series also discussed, which was taken on a high plateau in the Pyrenees, there was practically no shelter from wind at all. Modern practice proceeds on the understanding that moving air is almost essential, but that efficient shelter is to be provided from the direct action of the wind, hence the use of screens such as the Stevenson pattern. In some forms artificial wind is provided. Thus Svensson's work was done with an Assmann ventilated psychrometer,[1] a steady flow of air being maintained by a centrifugal pump through a cylinder containing the two thermometers. Different workers appear to hold very diverse opinions as to the reliability of the Assmann method.[2] As a control method Svensson used the Sonden hygrometer,[3] which measures the increase of volume produced on saturating the air. On this account, and also because of the different results of his two papers, Hazen severely criticises the whole investigation. We have not ourselves had an opportunity of using the Sonden instrument.

It is highly probable, therefore, that the apparent diminution of f in the observations discussed by Ekholm is due to a failure to comply with the conditions requisite for accurate use of the psychrometer, and this supposition is strongly borne out by the results of our observations, taken under more suitable conditions. Such a failure may occur occasionally under any circumstances, but, being of the nature of an accidental rather than a systematic error, it will be properly included among the casual variations of the single constant A. If, on the other hand, the diminution be due to hygroscopic action, it will be

1. Assmann, Meteor. Zeitschr., 1891, p. 15.
2. Hazen, "Psychrometer Studien," Meteor. Zeitschr., 1896, p. 275. "Das Problem des Psychrometers," 1899, p. 261.
3. Sonden, "Ein Neues Hygrometer," Meteor. Zeitschr., 1892, p. 81.

of a permanent nature, remaining more or less equal in all observations. Hence we have a means of directly testing the theory. If it be true, the observed value of t' will depend on the material used, and will always be too high. This will necessitate a correction to t' before inserting it in the ordinary psychrometrical tables, which are calculated for the standard case of $\eta = 1$. A short provisional table is given by Ekholm as an example of the corrections which would have to be applied. If, therefore, several wet-bulb thermometers covered with different materials be simultaneously observed, their readings should differ by amounts deducible from the values of η for each. As a rough indication of the extent of the differences to be expected at ordinary temperatures, consider the formula in the approximate form

$$x = \eta f - \tfrac{1}{2}(t - t').$$

Using differential notation, suppose η to change by $d\eta$, and therefore t' by dt' and f by df, while x and t remain unaltered. Then

$$0 = fd\eta + \eta df + \tfrac{1}{2}dt'$$

or, putting $\eta = 1$ in the second term as a close approximation,

$$-fd\eta = df + \tfrac{1}{2}dt'$$

For linen and cotton, $d\eta$ is given by Ekholm as 0.056 ; near 0° C. we have $df = \frac{1}{3}dt'$ very nearly, whence $dt' = 0.31°$ neglecting sign. Similarly for linen and silk, $d\eta$ is given as 0.034 ; in this way we can draw up a table such as the following:—

				Linen-cotton.		Linen-silk.
t' about 0°	-	$df = \frac{1}{3}dt'$	-	$dt' = 0.31°$	-	0.19°
6°	-	$\frac{1}{2}$	-	.39°	-	.24°
12°	-	$\frac{2}{3}$	-	.50°	-	.31°
18°	-	1	-	.58°	·	.35°

Such differences are, of course, easily recognisable if they exist.

Thermometer-screen and Preliminary Work.

In carrying out this comparison it was thought advisable to restrict ourselves to natural atmospheric conditions, since they are the ones which are of practical importance. To examine the effect of different influences on the psychrometer, artificial regulation may be best, but the applicability of the results in practice must be tested under the conditions in which meteorological observations are usually taken. The ordinary types of thermometer-screen were not, however, suitable for the purpose, since it was desired not only to use several wet-bulb thermometers, but also to directly determine the humidity by another method. For these reasons a new screen was designed by one of us (E.F.J.L.), which is depicted in Plate XLVIII., looking respectively N.W. and S.W. towards the screen. It is large enough to allow the observer to be inside and yet well away from the instruments, and by means of a sliding bench it is always possible to keep to leeward of them, so that the effects of radiation from the person are as small as possible. Full protection from the sun's direct rays is afforded, and a double roof prevents heating from that direction, the air circulating freely between the roofs. The force of the wind is broken by louvring as shown, the ventilation being always ample. The dimensions of the screen internally are 65 in. (165 cms.) each way, the height running from 65 to $87\frac{1}{2}$ in. (165 to 222 cms.). A trolley traverses the whole length, carrying a bench 48 in. (122 cms.) long, at a height of 50 in. (127 cms.) from the ground. The instruments are placed on this bench, which is adjusted to be out of the sun's rays and to windward of the observer. The outer roof is 12 in. (30.5 cms.) above the other.

By the kind permission of the University Council, and the generous provision by Professor Lyle of the necessary funds from the apparatus grant of the Natural Philosophy Department, the screen was set up in a suitable position in the grounds, the outlook being open on all sides, except for a wall distant 11 yards (10 metres) on the east. The prevailing winds are north and south, hence the screen was set facing south, the roof sloping downwards to the north.

Preliminary work was carried on by Mr. T. C Sutton, B.Sc., in 1909, with degree thermometers. Eleven thermometers of precisely similar size and shape were carefully compared and calibrated, of which seven were employed as wet-bulbs, and were covered respectively with silk, linen, wool (worsted), flannel, muslin, cotton-wool, and a mixed tussore fabric; the materials were first cleansed from grease, starch and other impurities by boiling in soda and distilled water. The whole eleven thermometers were suspended in a row above the bench, with the four dry-bulbs in the positions 1, 4, 8 and 11, the row being at right angles to the wind. Observations were only taken when the four dry-bulbs showed the same steady temperature, since then the intermediate wet-bulbs could be assumed to be under identical conditions. While the readings of the wet-bulbs were being taken, an observation of the dew-point was taken with a Regnault hygrometer. In all some sixty (60) observations were made, and Mr. Sutton reports that a striking feature is the consistency with which the various wet-bulbs rise and fall together with fluctuations in the humidity, etc. He also states: "In no case was there the slightest sign of any difference of temperature between the bulbs, so long as the water used was pure, and the terminal dry-bulb thermometers indicated the same temperature." These results showed that the difference, if any, was decidedly less than 0.1° C for dry temperatures ranging between 17° and 41.7° C. The next year, Mr. Sutton, being in England, made some more observations in a sheltered spot in the open at temperatures between —4° and 10°, at Twickenham, Middlesex. The differences in this case were certainly less than 0.2°, the limit of accuracy of the thermometers employed.

We were thus encouraged to proceed with the work in earnest this year. The values of the constant A obtained by Mr. Sutton showed a marked dependence on the velocity of the wind, suggesting some slight alterations in the construction of the screen which were carried out before beginning the new series of observations.

Apparatus and Procedure.

Three wet-bulb thermometers were used, all of Jena glass, graduated in tenths of a degree Centigrade, and with cylindrical bulbs. The materials were (1) silk, (2) linen, (3) cotton. Unfortunately the silk-covered one was broken after the work had been proceeding for some time, and before calibration had been completely carried out; it was replaced by another one, whose bulb was similar but shorter, no. 4 in the tables. The materials were cut from samples of fine Japanese silk, fine linen cambric, and book muslin of open texture; the pieces were thoroughly cleansed by boiling three times in succession, in caustic soda solution and in distilled water alternately. They were then sewn firmly round the bulbs, one thickness only, the rest hanging below.

The dry thermometer was also of Jena glass, similar to the others; it formed part of the condensation hygrometer used as a control. This was a Regnault double hygrometer, consisting of two precisely similar thimbles, one of which contains ether, and through both of which a steady current of air is drawn by an aspirator. Exception has sometimes been taken to the Regnault hygrometer on the ground that the silvered surfaces cannot be kept polished, being attacked by the gases of the air. This difficulty is avoided by using nickeled surfaces, which are found to keep a splendid polish until the nickel wears off. The curved surface is a slight disadvantage, but experience shows that it is of no consequence. The dew-point thermometer was of English glass, graduated in fifths of a degree Centigrade. The following table gives the dimensions and range of the several thermometers:—

	Bulb. Length.	Bulb. Diameter.	10 degree Length.	Range.
1	4.5 cms.	0.52 cm.	4.40 cms.	−7° to 50°C.
2	4.0	.48	4.64	−4° ,,
3	4.0	.49	4.45	−5° ,,
4	2.9	.48	4.34	−8° ,,
Dry	3.6	.50	4.60	−5° ,,
Dew-pt.	3.7	.52	4.67	−12° to 46°

They were calibrated by direct comparison with a standard Negretti and Zambra, No. 65043, tested at Kew in March, 1888. The scale errors of this standard were all zero, except —0.1° at 100° C. The zero error, determined immediately before the calibration, was 0.2°. The scale errors of the other thermometers were in all cases small, those of the dew-point thermometer being practically zero throughout. Nos. 2 and 3 were in agreement over the greater part of the scale. All temperatures were recorded to the nearest twentieth of a degree; owing to the smallness of the spaces between the graduations and the unavoidable slight fluctuations during the course of an experiment, any nearer approximation gives only an apparent increase of accuracy. Moreover, tables of vapour-pressure at present differ from one another by amounts comparable even to the tenth of a degree. The table actually employed was that obtained by Broch by reduction of Regnault's observations, as given in the Physical Tables, Smithsonian Miscellaneous Collections, vol. 35.

Each observation involved the following procedure:—The three wet-bulb thermometers were suspended with their coatings dipping into water, and left to attain equilibrium with the surrounding air. Meanwhile the barometer and attached thermometer were read. Then the Regnault hygrometer was set up on the bench, which was adjusted to suit the wind; the wet-bulbs were thoroughly wetted by raising the water-vessel, which was then removed. The aspirator was started, and the wet-bulbs watched; after some fluctuation a minimum steady temperature was attained, which was noted. The dry bulb was immediately read, and the dew-point determined as rapidly as possible; since the temperature of the ether had been lowered while the wet-bulbs were being watched, the whole set of observations could usually be obtained in well under a minute. (The dry bulb is found to attain a steady temperature more quickly if it is dipped in the water for a few moments and then carefully dried before placing in the thimble; the current of air drawn past it ensures that it is always in contact with the outside air. At the beginning of the series an extra thermometer was hung outside with the wet-bulbs, and showed no divergence from that in the hygrometer). The direction and force of the wind were also noted—i.e., whether light, strong, etc. As far as possible, observations were taken twice a day, at about 11 a.m. and 3 p.m. Those here tabulated were taken between May 8 and July 4 inclusive.

Observations and Results.

The complete list of corrected readings of the wet-bulbs is given in Table I., with the differences between them. The coatings are respectively (2) linen, (3) cotton, (4) silk. The first 52 observations of the silk-covered thermometer are not given, since they could not be properly corrected; the preliminary rough comparison which was carried out was sufficient, nevertheless, to show that they differed in no case from the others by more than 0.1° or 0.2°.

TABLE I.

No.	2	3	4	2—3	2—4
1	12.65	12.55		0.10	
2	12.35	12.25		.10	
3	11.25	11.25		0	
4	11.85	11.85		0	
5	12.95	12.95		0	
6	11.75	11.75		0	
7	11.85	11.85		0	
8	10.15	10.20		—.05	
9	9.65	9.70		—.05	
10	11.45	11.35		.10	
11	13.05	12.95		.10	
12	12.25	12.25		0	
13	14.95	15.00		—.05	
14	15.20	15.20		0	
15	14.30	14 30		0	
16	11.75	11.85		—.10	
17	11.35	11.35		0	
18	11.15	11.15		0	
19	11.15	11.05		.10	
20	12.95	12.95		0	
21	12.65	12.60		.05	
22	12.05	12.00		.05	
23	11.85	11.75		.10	
24	10.55	10.50		.05	
25	11.25	11.20		.05	
26	6.25	6.20		.05	
27	5.45	5.40		.05	

No.	2	3	4	2—3	2—4
28	8.85	8.85		0	
29	8.95	9.00		—.05	
30	9.45	9.50		—.05	
31	10.15	10.20		—.05	
32	8.75	8.90		—.15	
33	8.95	8.80		.15	
34	8.45	8.50		—.05	
35	8.15	8.10		.05	
36	5.50	5.45		.05	
37	5.65	5.55		.10	
38	7.85	7.80		.05	
39	8.15	8.10		.05	
40	7.75	7.80		—.05	
41	9.30	9.35		—.05	
42	5.20	5.15		.05	
43	5.85	5.90		—.05	
44	6.75	6.80		—.05	
45	7.35	7.40		—.05	
46	11.05	11.05		0	
47	11.15	11.10		.05	
48	10.05	10.10		—.05	
49	10.25	10.20		.05	
50	10.05	10.00		.05	
51	8.65	8.60		.05	
52	9.95	10.00		—.05	
53	8.85	8.90	8.85	—.05	0
54	9.15	9.20	9.15	—.05	0
55	10.15	10.20	10.05	—.05	.10
56	8.55	8.60	8.60	—.05	—.05
57	8.75	8.85	8.70	—.10	.05
58	7.55	7.50	7.40	.05	.15
59	6.45	6.60	6.45	—.15	0
60	6.55	6.60	6.65	—.05	—.10
61	9.45	9.40	9.40	.05	.05
62	8.85	8.80	8.75	.05	.10
63	7.75	7.60	7.60	.15	.15

Considering first nos. 2 and 3, linen and cotton, which differ most widely according to Ekholm's theory, this table shows that the differences are as often positive as negative—viz., out of 63 observations. 27 differences are positive, 23 are negative, and 13 are zero. They are distributed quite arbitrarily, without any reference to the values of t or t'. The largest individual difference is 0.15°, and this is twice positive and twice negative. The mean difference is 0.007°, which is certainly negligible, and the mean without regard to sign is only 0.05°, which is the observable limit.

For linen and silk there are only 11 observations, but the tendency to similar results is easily seen. The mean difference in this case is 0.04°, and the mean without regard to sign 0.07°. By comparison with the values to be expected from Ekholm's formula, it is evident that these are of the nature of accidental variations, and have no connection with any physical fact.

In Table II. are given the observations as a whole. The values under the heading t' are the means, where necessary, of the several wet-bulb readings; the pressure is given reduced to 0° C.

Table II.

No.	t	t'	Dew-point	B	Wind
1	14.75	12.60	11.05	763.8	Calm.
2	16.80	12.30	7.75	761.6	Calm.
3	15.85	11.25	6.45	760.2	Calm.
4	13.45	11.85	11.15	760.9	Gusty S.
5	15.65	12.95	11.75	759.2	Calm.
6	14.15	11.75	10.50	761.3	Strong S.E.
7	13.55	11.85	11.05	761.5	Light S.
8	13.55	10.20	7.55	766.2	Light S.
9	12.50	9.70	5.95	765.1	Light S.
10	14.05	11.40	9.10	767.3	Calm.
11	17.20	13.00	8.90	766.7	Calm.
12	14.65	12.25	10.50	766.6	Calm.
13	19.35	15.00	11.35	763.4	Light N.
14	18.10	15.20	12.85	758.6	N. breeze.
15	17.40	14.30	11.25	757.5	N. breeze.
16	16.70	11.80	8.10	756.9	N. breeze.
17	14.20	11.35	9.00	758.7	Strong N.
18	11.90	11.15	10.95	761.5	Fresh S.

No.		t		t'		Dew-point		B		Wind
19	-	12.40	-	11.10	-	10.50	-	759.8	-	S. breeze.
20	-	14.25	-	12.95	-	11.45	-	760.5	-	Calm.
21	-	14.45	-	12.60	-	11.30	-	761.0	-	Light S.
22	-	15.35	-	12.00	-	10.00	-	760.9	-	Fresh N.
23	-	14.35	-	11.80	-	9.10	-	759.1	-	Strong N.
24	-	12.80	-	10.50	-	8.60	-	756.0	-	Strong N.
25	-	13.65	-	11.20	-	9.15	-	754.0	-	Strong N.
26	-	7.35	-	6.20	-	4.95	-	761.4	-	Strong S.
27	-	8.55	-	5.40	-	1.85	-	760.4	-	Light S.
28	-	11.10	-	8.85	-	5.90	-	757.6	-	S.W. breeze.
29	-	11.80	-	9.00	-	5.85	-	757.4	-	Strong S.W.
30	-	11.20	-	9.50	-	7.75	-	761.8	-	Light S.
31	-	11.10	-	10.20	-	8.55	-	761.4	-	Calm.
32	-	11.80	-	8.80	-	4.60	-	766.4	-	Calm.
33	-	12.95	-	8.90	-	4.55	-	764.6	-	N. breeze.
34	-	11.80	-	8.50	-	5.75	-	763.9	-	N. breeze.
35	-	11.20	-	8.10	-	5.05	-	762.1	-	N. breeze.
36	-	7.65	-	5.50	-	3.30	-	757.5	-	Calm.
37	-	8.95	-	5.60	-	2.30	-	756.6	-	Light S.W.
38	-	10.25	-	7.80	-	2.95	-	762.2	-	Light N.W.
39	-	11.60	-	8.10	-	3.10	-	760.3	-	Light N.W.
40	-	9.65	-	7.80	-	5.95	-	754.3	-	Strong N.W.
41	-	11.20	-	9.30	-	7.15	-	750.8	-	Strong N.W.
42	-	8.45	-	5.20	-	0.80	-	747.0	-	Strong N.W.
43	-	9.25	-	5.90	-	1.00	-	746.8	-	Strong N.W.
44	-	9.45	-	6.80	-	4.15	-	754.3	-	S.W. breeze.
45	-	9.75	-	7.40	-	5.10	-	756.1	-	Light S.W.
46	-	12.00	-	11.05	-	10.10	-	766.1	-	Light S.W.
47	-	12.10	-	11.10	-	10.20	-	765.9	-	Light S.
48	-	10.90	-	10.10	-	8.95	-	770.4	-	Calm.
49	-	12.50	-	10.20	-	7.50	-	769.5	-	Calm.
50	-	12.40	-	10.00	-	7.25	-	767.4	-	Light N.W.
51	-	10.90	-	8.60	-	6.90	-	766.2	-	Light N.
52	-	12.00	-	10.00	-	7.95	-	763.3	-	Light N.
53	-	12.10	-	8.85	-	6.10	-	769.1	-	N. breeze.
54	-	12.35	-	9.15	-	5.95	-	768.2	-	Light N.
55	-	12.80	-	10.15	-	7.35	-	770.1	-	Calm.
56	-	10.85	-	8.60	-	7.30	-	767.7	-	Light N.

No.	t	t'	Dew-point	B	Wind
57	11.30	8.75	6.35	764.0	N. breeze.
58	10.85	7.50	3.65	764.1	Variable light.
59	10.95	6.50	1.20	763.7	Light S.W.
60	9.35	6.60	2.55	768.4	Light N.
61	9.75	9.40	9.00	769.0	Calm.
62	10.65	8.80	6.75	768.7	Calm.
63	9.65	7.65	3.95	774.1	Calm.

Applying the formula,

$$x = f - AB\,(t - t'),$$

the vapour-pressures being taken from Broch's tables, the values of A are calculated. The mean is 0.0007228, with a probable error of 0.0000175, or, approximately,

$$A = (72 \pm 2) \times 10^{-5}$$

The individual values of A vary rather widely, but all have been included, since the comparison of the wet-bulbs is not affected by other errors, such as a lack of precision in the indication of the Regnault hygrometer. Some of the largest discrepancies have been traced with great probability to an error of reading amounting to half a degree. The tendency to such a mistake was always present, owing to the method of marking of the thermometers, and its occurrence was actually detected in later observations, when it was watched for. It was not thought advisable, however, to make any tentative corrections of this nature. The outstanding errors being few, and both positive and negative, will not affect the mean value by much. It is somewhat high, compared with other observers' results, but this may be partly due to the fact that the values of $t - t'$ are all small, none being above 5°, although the dry temperatures range from 7.35° to 19.35°. As is seen in the next table, the values of the humidity are correspondingly fairly high throughout, being never less than 50 per cent.

With this mean value of A the corresponding values of x were calculated, and are included in Table III. under $\bar{x}$. The humidities calculated from x and $\bar{x}$ are also tabulated as r and $\bar{r}$, with the difference, or errors, $\Delta x = x - \bar{x}$ and $\Delta r = r - \bar{r}$.

TABLE III.

No.	x	x	Δx	r	r	Δr
1	9.80	9.66	.14	78.59	77.47	1.12
2	7.86	8.16	—.30	55.31	57.42	—2.11
3	7.19	7.40	—.21	53.74	55.31	—1.57
4	9.87	9.45	.42	86.05	82.39	3.66
5	10.27	9.62	.65	77.75	72.83	4.92
6	9.45	8.95	.50	78.75	74.58	4.17
7	9.80	9.39	.41	84.92	81.37	3.55
8	7.75	7.40	.35	67.16	64.12	3.04
9	6.95	7.41	—.46	64.47	68.74	—4.27
10	8.61	8.56	.05	72.23	71.81	0.42
11	8.49	8.81	—.32	58.23	60.43	—2.20
12	9.45	9.28	.17	76.27	74.90	1.37
13	10.00	10.27	—.27	59.95	61.57	—1.62
14	11.03	11.25	—.22	71.49	72.91	—1.42
15	9.93	10.42	—.49	67.28	70.59	—3.31
16	8.05	7.62	.43	56.97	53.93	3.04
17	8.55	8.44	.11	71.01	70.10	0.91
18	9.74	9.46	.28	93.93	91.22	2.71
19	9.45	9.13	.32	88.23	85.25	2.98
20	10.06	10.39	—.33	83.28	86.01	—2.73
21	9.97	9.83	.14	81.52	80.37	1.15
22	9.14	8.59	.55	70.52	66.28	4.24
23	8.61	8.90	—.29	70.81	73.19	—2.38
24	8.32	8.19	.13	75.70	74.52	1.18
25	8.63	8.56	.07	74.27	73.66	0.61
26	6.49	6.44	.05	84.95	84.29	0.66
27	5.22	4.96	.26	62.97	59.83	3.14
28	6.93	7.23	—.30	70.43	73.48	—3.05
29	6.90	7.02	—.12	66.99	68.16	—1.17
30	7.85	7.90	—.05	79.29	79.80	—0.51
31	8,29	8.76	—.47	84.25	89.02	—4.77
32	6.33	6.77	—.44	61.46	65.73	—4.27
33	6.31	6.25	.06	56.85	56.31	0.54
34	6.86	6.45	.41	66.60	62.62	3.98
35	6.53	6.37	.16	65.96	64.31	1.65
36	5.78	5.55	.23	74.01	71.06	2.95
37	5.39	4.95	.44	63.26	58.10	5.16

No.	x	$\bar{x}$	Δx	r	$\bar{r}$	Δr
38	5.64	6.53	—.89	60.71	70.29	—9.58
39	5.70	6.13	—.43	56.10	60.34	—4.24
40	6.95	6.87	.08	77.83	76.93	0.90
41	7.55	7.69	—.14	76.26	77.68	—1.42
42	4.84	4.84	.00	58.74	58.74	0.00
43	4.91	5.11	—.20	56.50	58.80	—2.30
44	6.13	5.92	.21	69.58	67.20	2.38
45	6.56	6.39	.17	72.97	71.08	1.89
46	9.20	9.27	—.07	88.21	88.88	—0.67
47	9.26	9.29	—.03	88.19	88.48	—0.29
48	8.52	8.75	—.23	87.84	90.21	—2.37
49	7.73	7.98	—.25	71.71	74.03	—2.32
50	7.59	7.81	—.22	70.87	72.92	—2.05
51	7.42	7.05	.37	76.42	72.61	3.81
52	7.96	8.04	—.08	76.32	77.09	—0.77
53	7.02	6.65	.37	66.86	63.33	3.53
54	6.95	6.85	.10	65.14	64.20	0.94
55	7.65	7.75	—.10	69.71	70.52	—0.81
56	7.62	7.07	.55	78.80	73.11	5.69
57	7.15	6.99	.16	71.72	70-11	1.61
58	5.92	5.88	.04	61.22	60.81	0.41
59	4.98	4.76	.22	51.13	48.87	2.26
60	5.48	5.73	—.25	65.28	68.26	—2.98
61	8.55	8.59	—.04	95.11	95.55	—0.44
62	7.34	7.40	—.06	76.94	77.57	—0.63
63	6.07	6.69	—.62	67.97	74.92	—6.95

In spite of the large variations of A from the mean, the errors of x and r are for the most part small. The probable error of a single observation of x is 0.22 mm. and of r 2.05 per cent. A few large errors of course appear, but as stated before, they are most likely due to errors of reading.

It remains to apply Ekholm's two-constant formula to the same observations, and see whether any improvement is effected in the agreement between observation and calculation. The values of η and A, as determined by least squares from the 63 observations, are 0.9974 and 0.0007062 respectively. That is, η is indistinguishable from unity to the order of accuracy which can be attained in such observations, and A is within